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Superconductivity in non-adiabatic systems with variable charge-carrier density

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Abstract. The paper presents original expressions for the temperature of the superconducting transition T_C as well as for the coefficient of the isotope effect α in non-adiabatic superconductors with strong electron correlations and variable charge-carrier density. The equation for the cut-off momentum Q_c due to strong electron correlations is also derived. The dependence of the quantities T_C, α and Q_c on the charge-carrier density n in quasi-two-dimensional and three-dimensional systems is studied. At the point of its maximum, T_C is shown to easily reach the values characteristic of high-T_C materials because of non-adiabaticity. The quantity α depends strongly on the theory parameters and achieves values in the range 0.2–0.4. It is found that the field of existence of superconductivity and the behaviour of $Q_c(n)$ are determined by the dimensionality of the system.

1. Introduction

In contemporary low-temperature physics, describing the properties of high-T_C materials is extremely complicated. They may be accounted for by the existence in the system of strong electron correlations induced by the Coulomb interaction between electrons, and of strong electron–phonon interaction. A number of approaches to this task are reviewed in, for example, [1]. It is worth noting here that dielectric, magnetic and superconducting phase transitions may occur in these materials. Therefore, to build a theory of these transitions taking account of the above-mentioned interactions is next to impossible, unless radical simplifications are introduced. Nevertheless, at a certain density of charge carriers it is noted that a metallic state is induced in these materials. In the metallic state, electron states are strongly modified, but not destroyed by correlations. So, transition to the superconducting state accompanied by the formation of Cooper pairs (the BCS scenario) or of localized pairs (the scenario of Schafroth) may occur. Hence it is of undoubted interest to study superconducting properties taking into account some of the striking features characteristic of high-T_C materials. Among them are: a layered structure (multiband character of the system), the existence of van Hove–Lifshitz singularities in the electron-state density and a variable density of charge carriers [2–9]. Also, oxide ceramics (organic superconductors and fullerenes) have low values of the Fermi energy (E_F) that can be of the same order as the Debye energy (ω_D). As a result there is violation of the Migdal theorem [10], used in the theory of ordinary superconductors if $\omega_D \ll E_F$. The same will be true for the systems with not only the electron–phonon pairing mechanism but also the arbitrary electron–boson one, which leads to a superconducting state if the Fermi energy and the characteristic boson frequency are quantities of the same order. So, if the materials mentioned above are in the metallic state with the arbitrary pairing-mediated mechanism, we have to go beyond the BCS–Bogoliubov theory through taking account of vertex and intersecting diagrams for the electron–boson interaction, which corresponds to additional multiparticle effects.

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Estimations of the vertex contributions P_V and P_c, and of the temperature of the superconducting transition T_C with account taken of the effects of non-adiabaticity (when we go beyond the framework of the Migdal theorem) are given in some papers [11–15]. There, a three-dimensional system with a symmetrical filling of the energy bands is discussed, and both the Migdal parameter $m = \omega_D/E_F$ and the cut-off parameter of the electron–phonon interaction $Q_c = q_c/2k_F$ are introduced. The introduction of the latter parameter is based on studies of the influence of strong electron correlations on the electron–phonon interaction [16, 17]. In those papers, low values of Q_c ($Q_c \ll 1$), characteristic of materials with strong electron correlations, are shown to favour both the inducing of positive values of the vertex functions and increase of the temperature of the superconducting transition. Even at intermediate values of interaction couplings of the electron–phonon interaction, $\lambda \sim 0.5$–1, high values of T_C can be obtained. It is possible to theoretically describe the properties of the systems for given values of the Fermi energy (half-filling of the energy bands).

Due to the strong anisotropy of the oxide superconductors, the concentration of the charge carriers changes with variations of the oxygen content and on introduction of a non-isovalent impurity.

In the present paper the main aim is to study the dependence of both the temperature of the superconducting transition T_C and the coefficient of the isotope effect α as functions of the charge-carrier density. This study will be made for two-dimensional (or quasi-two-dimensional) and three-dimensional systems with non-adiabatic effects taking into account strong electron correlations. It is shown that the imaginary parts of the vertex functions P_V and P_c (which are omitted in the above-mentioned papers) play an essential role in determining the field of existence of superconductivity and allow us to derive an equation for the quantity Q_c, which in [11–15] is considered as a theoretical parameter. The quantity Q_c depends on the charge-carrier density, and the character of this dependence is determined by the dimensionality of the system. If the energy band is symmetrically filled [12, 13], superconductivity in the system is possible only at $Q_c = 0$. Therefore, the quantity Q_c cannot be considered as a theoretical parameter taking various values for this system.

The electron–phonon interaction in the systems considered is undoubtedly playing an essential role. In addition to this fact, as regards the thermodynamic properties of the system the suggested model is valid for an arbitrary electron–boson interaction that causes superconductivity in a metallic state if the transferred momentum is small ($q \ll 2k_F$). So, in our opinion this fact is significant, because a pairing mechanism in high-T_C materials has not been established yet.

This paper has the following structure. Section 2 presents equations for the mass operators and the Green functions. Also, the vertex functions P_V and P_c for two-dimensional and three-dimensional systems with strong electron correlations are calculated. Section 3 contains analytical expressions for the temperature of the superconducting transition T_C and the equation for determining the quantity Q_c, dependent on the charge-carrier density. The last section discusses numerical calculations, and graphical dependences of the quantities Q_c, T_C and α on the charge-carrier density. Also the main results of the paper are discussed there.

2. Basic equations; calculation of vertex functions

We start from the Frohlich Hamiltonian, including an electron–phonon interaction with the interaction coupling g_{pp_1}, determined by the relationship [13]

$$g^2_{pp'} = g^2\gamma\theta(q_c - |\boldsymbol{p} - \boldsymbol{p}'|) \tag{1}$$

where q_c is a cut-off momentum of the electron–phonon interaction and γ is determined from the condition

$$\left\langle\left\langle g^2_{pp'}\right\rangle\right\rangle_{FS} = g^2. \tag{2}$$

Here $\langle\langle\cdots\rangle\rangle_{FS}$ denotes averaging over the Fermi surface. This choice of the interaction coupling is based on the results of studies [16,17] that demonstrate strong electron correlations cutting off the electron–phonon interaction at the momentum $q_c \ll 2p_F$.

We introduce first the temperature (Matsubara) one-electron Green functions:

$$G_{\beta\alpha}(xx') = -\left\langle T\psi_\beta(x)\psi^+_\alpha(x')\right\rangle \qquad F_{\beta\beta'}(xx') = -\left\langle T\psi_\beta(x)\psi_{\beta'}(x')\right\rangle \tag{3}$$

and then the phonon Green function;

$$D(xx') = -\left\langle T\varphi(x)\varphi(x')\right\rangle. \tag{4}$$

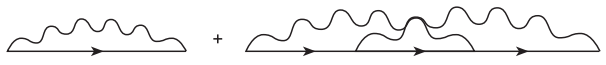
Here $x = (\boldsymbol{x}, \tau)$, $\psi_\alpha(x) = \mathrm{e}^{\tau H}\psi_\alpha(\boldsymbol{x})\mathrm{e}^{-\tau H}$, $\psi^+_\alpha(x) = \mathrm{e}^{\tau H}\psi^+_\alpha(\boldsymbol{x})\mathrm{e}^{-\tau H}$, H is a complete Hamiltonian of an electron–phonon system and $\langle\cdots\rangle$ implies averaging over the states of the system of interacting particles.

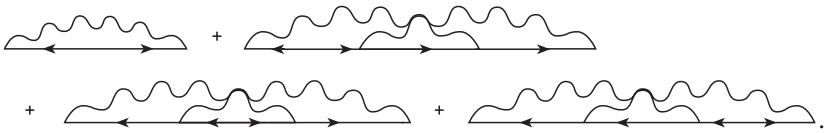
In expressions (3) and (4) we pass over to the interaction representation and use the perturbation theory [18] for the electron–phonon interaction. In this way, for the Green functions (3) in $\boldsymbol{p}$–Ω representation we derive the following expressions:

$$\begin{aligned} &G(\boldsymbol{p},\Omega) = -\frac{\mathrm{i}\Omega + \varepsilon_p + M(-\boldsymbol{p},-\Omega)}{A(\boldsymbol{p},\Omega)} \qquad F_{\uparrow\downarrow}(\boldsymbol{p},\Omega) = -\frac{\Sigma(\boldsymbol{p},\Omega)}{A(\boldsymbol{p},\Omega)} \\ &A(\boldsymbol{p},\Omega) = \left[\mathrm{i}\Omega - \varepsilon_p + M(\boldsymbol{p},\Omega)\right]\left[-\mathrm{i}\Omega - \varepsilon_p - M(-\boldsymbol{p},-\Omega)\right] + |\Sigma(\boldsymbol{p},\Omega)|^2. \end{aligned} \tag{5}$$

Here $\Omega = (2n+1)\pi T$, ε_p is the energy of an electron and M, Σ are the mass operators with vertex corrections and intersecting diagrams.

Near the temperature of the superconducting transition ($T \sim T_C$) we can restrict our consideration to terms linear in the operator Σ. In the diagram representation, we have

$M(\boldsymbol{p},\Omega) =$ 

$\Sigma(\boldsymbol{p},\Omega) =$  . (6)

In these diagrams, solid lines for M and Σ usually represent the electron Green functions (5) linearized in terms of the quantity Σ:

$$\begin{aligned} &G(\boldsymbol{p},\Omega) = \left[\mathrm{i}\Omega - \varepsilon_p - M(\boldsymbol{p},\Omega)\right]^{-1} \\ &F_{\uparrow\downarrow}(\boldsymbol{p},\Omega) = G(-\boldsymbol{p},-\Omega)\Sigma(\boldsymbol{p},\Omega)G(\boldsymbol{p},\Omega). \end{aligned} \tag{7}$$

The wavy line in (6) represents the phonon Green function:

$$D(\boldsymbol{p}-\boldsymbol{p}_1, \Omega-\Omega_1) = -g^2_{pp_1}\frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2} \tag{8}$$

with the simple Einstein spectrum ω_0. Expressions (6) can be derived in the form

$$M(\boldsymbol{p},\Omega) = \frac{1}{\beta V}\sum_{\boldsymbol{p}_1\Omega_1} V_N(pp_1)G(\boldsymbol{p}_1,\Omega_1) \tag{9}$$

$$\Sigma(\boldsymbol{p},\Omega)=\frac{1}{\beta V}\sum_{\boldsymbol{p}_1\Omega_1}V_S(pp_1)F_{\uparrow\downarrow}(\boldsymbol{p}_1,\Omega_1)$$
$$=\frac{1}{\beta V}\sum_{\boldsymbol{p}_1\Omega_1}V_S(pp_1)G(\boldsymbol{p}_1,\Omega_1)G(-\boldsymbol{p}_1,-\Omega_1)\Sigma(\boldsymbol{p}_1,\Omega_1) \tag{10}$$

where

$$V_N(pp_1)=-D(\boldsymbol{p}-\boldsymbol{p}_1,\Omega-\Omega_1)\,[1+\lambda P_V(\boldsymbol{pp}_1,\Omega\Omega_1)] \tag{11}$$
$$V_S(pp_1)=-D(\boldsymbol{p}-\boldsymbol{p}_1,\Omega-\Omega_1)[1+\lambda P_V(\boldsymbol{pp}_1,\Omega\Omega_1)$$
$$+\lambda P_V(\boldsymbol{pp}_1,-\Omega-\Omega_1)+\lambda P_c(\boldsymbol{pp}_1,\Omega\Omega_1)] \tag{12}$$

where $\lambda=g^2N_0$, N_0 is the density of electron states on the Fermi surface, and P_V and P_c are the vertex functions determined by the relationships

$$P_V(\boldsymbol{pp}_1,\Omega\Omega_1)=\frac{1}{\beta V}\frac{\gamma}{N_0}\sum_{\boldsymbol{p}_2\Omega_2}\theta(q_c-|\boldsymbol{p}-\boldsymbol{p}_2|)\frac{\omega_0^2}{(\Omega-\Omega_2)^2+\omega_0^2}$$
$$\times G(\boldsymbol{p}_2,\Omega_2)G(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{p},\Omega_1+\Omega_2-\Omega)$$
$$P_c(\boldsymbol{pp}_1,\Omega\Omega_1)=\frac{1}{\beta V}\frac{\gamma}{N_0}\sum_{\boldsymbol{p}_2\Omega_2}\theta(q_c-|\boldsymbol{p}-\boldsymbol{p}_2|)\frac{\omega_0^2}{(\Omega-\Omega_2)^2+\omega_0^2} \tag{13}$$
$$\times G(\boldsymbol{p}_2-\boldsymbol{p}-\boldsymbol{p}_1,\Omega_2-\Omega-\Omega_1)G(\boldsymbol{p}_2,\Omega_2).$$

The first terms in the effective interactions (11) and (12) correspond to adiabatic contributions; the rest correspond to diagrams with intersecting lines for the electron–phonon interaction.

2.1. Two-dimensional systems

We introduce a quadratic dispersion law for the electron energy:

$$\varepsilon_p=\frac{p_x^2+p_y^2}{2m}-\mu. \tag{14}$$

In this case $\gamma=\pi/Q_c$.

Then we insert the Green function (7) in a zeroth approximation for the electron–phonon interaction in the expression (13) and go from summation over $\boldsymbol{p}_2$, Ω_2 to integration according to the formula

$$\frac{1}{\beta V}\sum_{\boldsymbol{p}_2\Omega_2}F(\boldsymbol{p}_2,\Omega_2)=N_0\int_0^{2\pi}\frac{\mathrm{d}\varphi}{2\pi}\int_{-\mu}^{W-\mu}\mathrm{d}\varepsilon_{\boldsymbol{p}_2}\,\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\Omega_2\;F(\boldsymbol{p}_2,\Omega_2) \tag{15}$$

where $N_0=m/2\pi$, W is the halfwidth of the energy band, μ is the chemical potential.

Here we have introduced an electron–hole asymmetry in the integration over energy, and also assumed that $T_C\ll\omega_0$, which allows us to go to integration over frequency for $T_C\to 0$.

In accordance with (14), for low values of the transferred momentum $q=|\boldsymbol{p}_F-\boldsymbol{p}_{1F}|$ we have

$$\varepsilon_{\boldsymbol{p}_2+\boldsymbol{p}_1-\boldsymbol{p}}\approx\varepsilon_{\boldsymbol{p}_2}+EQ^2\frac{\varphi^2}{2}+EQ\sqrt{1-Q^2}\varphi$$
$$\varepsilon_{\boldsymbol{p}_2-\boldsymbol{p}_1-\boldsymbol{p}}\approx\varepsilon_{\boldsymbol{p}_2}+E(1-Q^2)\frac{\varphi^2}{2}-EQ\sqrt{1-Q^2}\varphi \tag{16}$$

where

$$E=4E_F\qquad Q=\frac{q}{2p_F}\qquad E_F=\frac{p_F^2}{2m}.$$

After carrying out the integration over frequency and energy in (13) and then integration over φ (taking into account its smallness), we have the vertex functions in the form

$$\begin{aligned} P_V(\boldsymbol{pp}_1, \Omega\Omega_1) &= P_V(QQ_c, \Omega\Omega_1) = \mathrm{Re}\, P_V(QQ_c, \Omega\Omega_1) + \mathrm{i}\,\mathrm{Im}\, P_V(QQ_c, \Omega\Omega_1) \\ P_c(\boldsymbol{pp}_1, \Omega\Omega_1) &= P_c(QQ_c, \Omega\Omega_1) = \mathrm{Re}\, P_c(QQ_c, \Omega\Omega_1) + \mathrm{i}\,\mathrm{Im}\, P_c(QQ_c, \Omega\Omega_1). \end{aligned} \tag{17}$$

Conserving the terms of the order Q^4 at $\mu > 2EQ_c^2$, we derive

$$\begin{aligned} \mathrm{Re}\, P_V(QQ_c, \Omega\Omega_1) &\approx \omega_0 B(\Omega\Omega_1) + \frac{\omega_0}{(\Omega-\Omega_1)^2}\left[A(\Omega\Omega_1) - (\Omega-\Omega_1)^2 B(\Omega\Omega_1)\right] \\ &\quad\times\left[1 - \frac{E^2}{(\Omega-\Omega_1)^2}\frac{4}{3}Q^2Q_c^2\right] + \frac{\omega_0 E}{(\Omega-\Omega_1)^2}C(\Omega\Omega_1)QQ_c \\ \mathrm{Re}\, P_c(QQ_c, \Omega\Omega_1) &\approx \omega_0 B(\Omega, -\Omega_1) + \frac{\omega_0}{(\Omega+\Omega_1)^2}\left[A(\Omega, -\Omega_1) - (\Omega+\Omega_1)^2 B(\Omega, -\Omega_1)\right] \\ &\quad\times\left[1 - \frac{E^2}{(\Omega+\Omega_1)^2}\left(\frac{4}{5}Q_c^4 - 2QQ_c^3 + \frac{4}{3}Q^2Q_c^2\right)\right] \\ &\quad + \frac{\omega_0 E}{(\Omega+\Omega_1)^2}C(\Omega, -\Omega_1)\left[\frac{2}{3}Q_c^2 - QQ_c\right] \end{aligned} \tag{18}$$

and

$$\begin{aligned} \mathrm{Im}\, P_V(QQ_c, \Omega\Omega_1) &\approx \omega_0 B_1(\Omega\Omega_1) + \frac{\omega_0}{(\Omega-\Omega_1)^2}\left[A_1(\Omega\Omega_1) - (\Omega-\Omega_1)^2 B_1(\Omega\Omega_1)\right] \\ &\quad\times\left[1 - \frac{E^2}{(\Omega-\Omega_1)^2}\frac{4}{3}Q^2Q_c^2\right] + \frac{\omega_0 E}{(\Omega-\Omega_1)^2}C_1(\Omega\Omega_1)QQ_c \\ \mathrm{Im}\, P_c(QQ_c, \Omega\Omega_1) &\approx \omega_0 B_1(\Omega, -\Omega_1) \\ &\quad + \frac{\omega_0}{(\Omega+\Omega_1)^2}\left[A_1(\Omega, -\Omega_1) - (\Omega+\Omega_1)^2 B_1(\Omega, -\Omega_1)\right] \\ &\quad\times\left[1 - \frac{E^2}{(\Omega+\Omega_1)^2}\left(\frac{4}{5}Q_c^4 - 2QQ_c^3 + \frac{4}{3}Q^2Q_c^2\right)\right] \\ &\quad + \frac{\omega_0 E}{(\Omega+\Omega_1)^2}C_1(\Omega, -\Omega_1)\left[\frac{2}{3}Q_c^2 - QQ_c\right] \end{aligned} \tag{19}$$

where

$$\begin{aligned} A(\Omega\Omega_1) &= \frac{\Omega-\Omega_1}{2}\left[2\arctan\frac{\Omega}{\omega_0} - \arctan\frac{\Omega}{W-\mu+\omega_0} - \arctan\frac{\Omega}{\mu+\omega_0}\right. \\ &\quad\left. + \arctan\frac{\Omega_1}{\mu+\omega_0} - 2\arctan\frac{\Omega_1}{\omega_0} + \arctan\frac{\Omega_1}{W-\mu+\omega_0}\right] \\ B(\Omega\Omega_1) &= -\frac{\mu+\omega_0}{2\left[(\mu+\omega_0)^2+\Omega_1^2\right]^2}\left[(\mu+\omega_0)^2 + 2\Omega_1^2 - \Omega\Omega_1\right] \\ &\quad - \frac{W-\mu+\omega_0}{2\left[(W-\mu+\omega_0)^2+\Omega_1^2\right]^2}\left[(W-\mu+\omega_0)^2 + 2\Omega_1^2 - \Omega\Omega_1\right] \\ C(\Omega\Omega_1) &= \frac{1}{4}\ln\frac{(W-\mu+\omega_0)^2+\Omega^2}{(\mu+\omega_0)^2+\Omega^2} - \frac{1}{4}\ln\frac{(W-\mu+\omega_0)^2+\Omega_1^2}{(\mu+\omega_0)^2+\Omega_1^2} \\ &\quad + \Omega_1(\Omega-\Omega_1)\frac{1}{2}\left[\frac{1}{(\mu+\omega_0)^2+\Omega_1^2} - \frac{1}{(W-\mu+\omega_0)^2+\Omega_1^2}\right] \end{aligned} \tag{20}$$

and

$$A_1(\Omega\Omega_1) = \frac{\Omega-\Omega_1}{4}\left\{\ln\frac{(W-\mu+\omega_0)^2+\Omega_1^2}{(\mu+\omega_0)^2+\Omega_1^2} - \ln\frac{(W-\mu+\omega_0)^2+\Omega^2}{(\mu+\omega_0)^2+\Omega^2}\right\}$$

$$\begin{aligned} B_1(\Omega\Omega_1) = {} & \frac{\Omega_1}{2}\left[\frac{1}{(\mu+\omega_0)^2+\Omega_1^2} - \frac{1}{(W-\mu+\omega_0)^2+\Omega_1^2}\right] \\ & + \frac{\Omega-\Omega_1}{4}\left[\frac{(\mu+\omega_0)^2-\Omega_1^2}{\left[(\mu+\omega_0)^2+\Omega_1^2\right]^2} - \frac{(W-\mu+\omega_0)^2-\Omega_1^2}{\left[(W-\mu+\omega_0)^2+\Omega_1^2\right]^2}\right] \end{aligned} \tag{21}$$

$$\begin{aligned} C_1(\Omega\Omega_1) = {} & \frac{1}{2}\Bigg[2\arctan\frac{\Omega}{\omega_0} - \arctan\frac{\Omega}{W-\mu+\omega_0} - \arctan\frac{\Omega}{\mu+\omega_0} \\ & + \arctan\frac{\Omega_1}{\mu+\omega_0} - 2\arctan\frac{\Omega_1}{\omega_0} + \arctan\frac{\Omega_1}{W-\mu+\omega_0}\Bigg] \\ & + \frac{\Omega-\Omega_1}{2}\left[\frac{\mu+\omega_0}{(\mu+\omega_0)^2+\Omega_1^2} + \frac{W-\mu+\omega_0}{(W-\mu+\omega_0)^2+\Omega_1^2}\right]. \end{aligned}$$

Then we perform an averaging over the Fermi surface, using the expressions (18) and (19), according to the formula

$$\begin{aligned} P_{V,c}(Q_c, \Omega\Omega_1) &= \frac{\pi}{Q_c}\left\langle\!\left\langle\theta(q_c - |\boldsymbol{p}-\boldsymbol{p}_1|)P_{V,c}(\boldsymbol{p}\boldsymbol{p}_1, \Omega\Omega_1)\right\rangle\!\right\rangle_{FS} \\ &= \frac{1}{Q_c}\int_0^{Q_c} \mathrm{d}Q\ P_{V,c}(QQ_c, \Omega\Omega_1). \end{aligned} \tag{22}$$

For the quantities Re $P_{V,c}(Q_c, \Omega\Omega_1)$ and Im $P_{V,c}(Q_c, \Omega\Omega_1)$ we obtain

$$\begin{aligned} \text{Re } P_V(Q_c, \Omega\Omega_1) = {} & \omega_0 B(\Omega\Omega_1) + \frac{\omega_0}{(\Omega-\Omega_1)^2}\left[A(\Omega\Omega_1) - (\Omega-\Omega_1)^2 B(\Omega\Omega_1)\right] \\ & \times \left[1 - \frac{E^2}{(\Omega-\Omega_1)^2}\frac{4}{9}Q_c^4\right] + \frac{\omega_0 E}{(\Omega-\Omega_1)^2}C(\Omega\Omega_1)\frac{Q_c^2}{2} \\ \text{Re } P_c(Q_c, \Omega\Omega_1) & \\ = {} & \omega_0 B(\Omega, -\Omega_1) + \frac{\omega_0}{(\Omega+\Omega_1)^2}\left[A(\Omega, -\Omega_1) - (\Omega+\Omega_1)^2 B(\Omega, -\Omega_1)\right] \\ & \times \left[1 - \frac{E^2}{(\Omega+\Omega_1)^2}\frac{11}{45}Q_c^4\right] + \frac{\omega_0 E}{(\Omega+\Omega_1)^2}C(\Omega, -\Omega_1)\frac{Q_c^2}{6} \end{aligned} \tag{23}$$

and

$$\begin{aligned} \text{Im } P_V(Q_c, \Omega\Omega_1) = {} & \omega_0 B_1(\Omega\Omega_1) + \frac{\omega_0}{(\Omega-\Omega_1)^2}\left[A_1(\Omega\Omega_1) - (\Omega-\Omega_1)^2 B_1(\Omega\Omega_1)\right] \\ & \times \left[1 - \frac{E^2}{(\Omega-\Omega_1)^2}\frac{4}{9}Q_c^4\right] + \frac{\omega_0 E}{(\Omega-\Omega_1)^2}C_1(\Omega\Omega_1)\frac{Q_c^2}{2} \\ \text{Im } P_c(Q_c, \Omega\Omega_1) & \\ = {} & \omega_0 B_1(\Omega, -\Omega_1) + \frac{\omega_0}{(\Omega+\Omega_1)^2}\left[A_1(\Omega, -\Omega_1) - (\Omega+\Omega_1)^2 B_1(\Omega, -\Omega_1)\right] \\ & \times \left[1 - \frac{E^2}{(\Omega+\Omega_1)^2}\frac{11}{45}Q_c^4\right] + \frac{\omega_0 E}{(\Omega+\Omega_1)^2}C_1(\Omega, -\Omega_1)\frac{Q_c^2}{6}. \end{aligned} \tag{24}$$

2.2. Three-dimensional systems

Here we will carry out calculations similar to those in [13], the only difference being that we take into account the variable charge-carrier density (electron–hole asymmetry) and do not

neglect the imaginary parts of the vertex functions P_V and P_c.

We perform the integration in expression (13) according to the relationship

$$\frac{1}{\beta V}\sum_{\boldsymbol{p}_2\Omega_2} F(\boldsymbol{p}_2,\Omega_2) = N_0\int_0^{2\pi}\frac{\mathrm{d}\varphi}{2\pi}\int_0^{\pi}\frac{\sin\alpha}{2}\,\mathrm{d}\alpha\int_{-\mu}^{W-\mu}\mathrm{d}\varepsilon_{p_2}\,\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\Omega_2\;F(\boldsymbol{p}_2,\Omega_2) \tag{25}$$

where $N_0 = mp_F/2\pi^2$. At low values of the transferred momentum $Q = q/2k_F$ we have [13]

$$\varepsilon_{\boldsymbol{p}_2+\boldsymbol{p}_1-\boldsymbol{p}} \approx \varepsilon_{p_2} + EQ\alpha\cos\varphi \qquad \varepsilon_{\boldsymbol{p}_2-\boldsymbol{p}_1-\boldsymbol{p}} \approx \varepsilon_{p_2} + E\frac{\alpha^2}{2} - EQ\alpha\cos\varphi \tag{26}$$

where α is angle between vectors $\boldsymbol{p}$ and $\boldsymbol{p}_2$.

Then we carry out calculations in the same way as in the two-dimensional case, averaging according to the relationship

$$\begin{aligned} P_{V,c}(Q_c,\Omega\Omega_1) &= \frac{1}{Q_c^2}\left\langle\!\!\left\langle\theta(q_c-|\boldsymbol{p}-\boldsymbol{p}_1|)P_{V,c}(\boldsymbol{p}\boldsymbol{p}_1,\Omega\Omega_1)\right\rangle\!\!\right\rangle_{FS} \\ &= \frac{2}{Q_c^2}\int_0^{Q_c} Q\,\mathrm{d}Q\;P_{V,c}(QQ_c,\Omega\Omega_1) \end{aligned} \tag{27}$$

and derive

$$\begin{aligned} \mathrm{Re}\,P_V(Q_c,\Omega\Omega_1) &= \frac{\omega_0 A(\Omega\Omega_1)}{(\Omega-\Omega_1)^2} - \frac{\omega_0E^2}{(\Omega-\Omega_1)^4}\left[A(\Omega\Omega_1)-(\Omega-\Omega_1)^2B(\Omega\Omega_1)\right]\frac{1}{2}Q_c^4 \\ \mathrm{Re}\,P_c(Q_c,\Omega\Omega_1) &= \frac{\omega_0 A(\Omega,-\Omega_1)}{(\Omega+\Omega_1)^2} - \frac{\omega_0E^2}{(\Omega+\Omega_1)^4}[A(\Omega,-\Omega_1) \\ &\quad -(\Omega+\Omega_1)^2B(\Omega,-\Omega_1)]\frac{11}{6}Q_c^4 + \frac{\omega_0E}{(\Omega+\Omega_1)^2}C(\Omega,-\Omega_1)Q_c^2 \end{aligned} \tag{28}$$

and

$$\begin{aligned} \mathrm{Im}\,P_V(Q_c,\Omega\Omega_1) &= \frac{\omega_0 A_1(\Omega\Omega_1)}{(\Omega-\Omega_1)^2} - \frac{\omega_0E^2}{(\Omega-\Omega_1)^4}\left[A_1(\Omega\Omega_1)-(\Omega-\Omega_1)^2B_1(\Omega\Omega_1)\right]\frac{1}{2}Q_c^4 \\ \mathrm{Im}\,P_c(Q_c,\Omega\Omega_1) &= \frac{\omega_0 A_1(\Omega,-\Omega_1)}{(\Omega+\Omega_1)^2} - \frac{\omega_0E^2}{(\Omega+\Omega_1)^4}[A_1(\Omega,-\Omega_1) \\ &\quad -(\Omega+\Omega_1)^2B_1(\Omega,-\Omega_1)]\frac{11}{6}Q_c^4 + \frac{\omega_0E}{(\Omega+\Omega_1)^2}C_1(\Omega,-\Omega_1)Q_c^2. \end{aligned} \tag{29}$$

Comparing the expressions (23) and (24) for the real and imaginary parts of the vertex functions for the two-dimensional superconductor to equations (28) and (29) for the three-dimensional case, we can clearly see that they differ by numerical coefficients in the terms containing Q_c^2 and Q_c^4.

3. Critical temperature and the equation for Q_c

For the effective electron–phonon interaction (11), (12), averaged over the Fermi surface, taking into account all of the calculations mentioned above, we have

$$\begin{aligned} \langle\langle V_N(\boldsymbol{p}\boldsymbol{p}_1)\rangle\rangle_{FS} &= \frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2}g^2\left[1+\lambda P_V(Q_c,\Omega,\Omega_1)\right] \\ \langle\langle V_S(\boldsymbol{p}\boldsymbol{p}_1)\rangle\rangle_{FS} &= \frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2}g^2[1+\lambda P_V(Q_c,\Omega,\Omega_1) \\ &\quad +\lambda P_V(Q_c,-\Omega,-\Omega_1)+\lambda P_c(Q_c,\Omega,\Omega_1)]. \end{aligned} \tag{30}$$

The expression for the Green function is given in the form

$$G(\boldsymbol{p},\Omega)=\frac{1}{\mathrm{i}\widetilde{\Omega}-\varepsilon_{\boldsymbol{p}}} \tag{31}$$

where

$$\widetilde{\Omega}=\Omega-\operatorname{Im}M(\Omega).$$

For the self-energy operators we derive

$$M(\Omega)=\frac{g^2}{\beta V}\sum_{\boldsymbol{p}_1\Omega_1}\frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2}\left[1+\lambda P_V(Q_c,0,\omega_0)\right]G(\boldsymbol{p}_1,\Omega_1) \tag{32}$$

$$\begin{aligned}\Sigma(\Omega)=\frac{g^2}{\beta V}\sum_{\boldsymbol{p}_1\Omega_1}\frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2}&[1+2\lambda P_V(Q_c,0,\omega_0)\\&+\lambda P_c(Q_c,0,\omega_0)]G(\boldsymbol{p}_1,\Omega_1)G(-\boldsymbol{p}_1,-\Omega_1).\end{aligned} \tag{33}$$

On the basis of (31), (32) and (15) or (25), after performing the summation over $\boldsymbol{p}_1$ and Ω_1, we obtain the following expression:

$$\widetilde{\Omega}=\Omega Z(\Omega)+Z_1 \tag{34}$$

where

$$\begin{aligned}&Z(\Omega)=1+\lambda_z\frac{1}{2}\left[\frac{\overline{W}-\overline{\mu}}{\overline{W}-\overline{\mu}+1}+\frac{\overline{\mu}}{\overline{\mu}+1}\right]+\lambda^2\operatorname{Im}P_V(Q_c,0,\omega_0)\frac{\Omega}{\omega_0}\frac{1}{4}\ln\frac{(\overline{W}-\overline{\mu})^2+1}{\overline{\mu}^2+1}\\&Z_1=\lambda^2\operatorname{Im}P_V(Q_c,0,\omega_0)\frac{\omega_0}{2}\ln\frac{(\overline{W}-\overline{\mu})^2+1}{\overline{\mu}^2+1}\qquad\overline{W}=\frac{W}{\omega_0}\qquad\overline{\mu}=\frac{\mu}{\omega_0}\\&\lambda_z=\lambda\left[1+\lambda\operatorname{Re}P_V(Q_c,0,\omega_0)\right].\end{aligned} \tag{35}$$

After performing the integration over energy in (33), with account taken of the electron–hole asymmetry and definitions (31) and (34), we obtain

$$\Sigma(\Omega)=\frac{1}{\beta}\sum_{\Omega_1}(\lambda_\Delta+\mathrm{i}\lambda^2\eta)\frac{\Sigma(\Omega_1)}{\Omega_1 Z}\left[\varphi_1(\Omega_1)+\mathrm{i}\varphi_2(\Omega_1)\right]\frac{\omega_0^2}{(\Omega-\Omega_1)^2+\omega_0^2} \tag{36}$$

where

$$\begin{aligned}\varphi_1(\Omega_1)=&\frac{1}{2}\left[\arctan\frac{W-\mu}{Z\Omega_1+Z_1}+\arctan\frac{\mu}{Z\Omega_1+Z_1}\right]\\&+\frac{1}{2}\left[\arctan\frac{W-\mu}{Z\Omega_1-Z_1}+\arctan\frac{\mu}{Z\Omega_1-Z_1}\right]\\\varphi_2(\Omega_1)=&\frac{1}{2}\ln\frac{(W-\mu)^2+(Z\Omega_1-Z_1)^2}{\mu^2+(Z\Omega_1-Z_1)^2}-\frac{1}{2}\ln\frac{(W-\mu)^2+(Z\Omega_1+Z_1)^2}{\mu^2+(Z\Omega_1+Z_1)^2}\end{aligned} \tag{37}$$

and

$$\lambda_\Delta=\lambda(1+\lambda\operatorname{Re}P_V(Q_c,0,\omega_0)+\lambda\operatorname{Re}P_c(Q_c,0,\omega_0)) \tag{38}$$

$$\eta=\operatorname{Im}[2P_V(Q_c,0,\omega_0)+P_c(Q_c,0,\omega_0)]. \tag{39}$$

From expressions (36) it follows that $\Sigma(\Omega)$ is a complex quantity. Thus we present it in the form $\Sigma=\Sigma_1+\mathrm{i}\Sigma_2$, taking into account the approximations that are used in both adiabatic [19] and non-adiabatic theory [13], and this changes (36) to the system of equations

$$\Sigma_1(\Omega)=\frac{\omega_0^2}{\Omega^2+\omega_0^2}V_1A_1^0-\frac{\omega_0^2}{\Omega^2+\omega_0^2}V_2A_2^0\qquad\Sigma_2(\Omega)=\frac{\omega_0^2}{\Omega^2+\omega_0^2}V_2A_1^0+\frac{\omega_0^2}{\Omega^2+\omega_0^2}V_1A_2^0 \tag{40}$$

where

$$V_1 = \lambda_\Delta - \lambda^2\eta\frac{\varphi_2(\omega_0)}{\varphi_1(\omega_0)} \qquad V_2 = \lambda_\Delta\frac{\varphi_2(\omega_0)}{\varphi_1(\omega_0)} + \lambda^2\eta \tag{41}$$

$$A_1^0 = \frac{1}{\beta}\sum_{\Omega_1}\frac{\omega_0^2}{\Omega_1^2+\omega_0^2}\frac{\varphi_1(\Omega_1)}{\Omega_1 Z}\Sigma_1(\Omega_1) \qquad A_2^0 = \frac{1}{\beta}\sum_{\Omega_1}\frac{\omega_0^2}{\Omega_1^2+\omega_0^2}\frac{\varphi_1(\Omega_1)}{\Omega_1 Z}\Sigma_2(\Omega_1). \tag{42}$$

Inserting (40) in (42) we derive

$$A_1^0 = A_1^0 V_1\xi_c - A_2^0 V_2\xi_c \qquad A_2^0 = A_1^0 V_2\xi_c + A_2^0 V_1\xi_c \tag{43}$$

where

$$\xi_c = \frac{1}{\beta}\sum_{\Omega_1}\frac{\omega_0^4}{(\Omega_1^2+\omega_0^2)^2}\frac{\varphi_1(\Omega_1)}{\Omega_1 Z}. \tag{44}$$

The temperature of the superconducting transition is determined from the consistency condition for the system of equations (43). We derive

$$(V_1^2 + V_2^2)\xi_c^2 - 2V_1\xi_c + 1 = 0. \tag{45}$$

Therefore,

$$\xi_c = \frac{V_1 \pm \sqrt{-V_2^2}}{V_1^2 + V_2^2}. \tag{46}$$

We obtain the real solution for the quantity T_C provided that $V_2 = 0$. This condition gives, to the accuracy of the first order in the non-adiabaticity,

$$\xi_c \simeq 1/V_1 \approx 1/\lambda_\Delta \tag{47}$$

$$\eta = \mathrm{Im}[2P_V(Q_c, \omega_0) + P_c(Q_c, \omega_0)] = 0. \tag{48}$$

After carrying out the integration over Ω_1 in (44) and inserting this expression in (47), we derive

$$T_C = \frac{1.13\omega_0\left[(\overline{W}-\overline{\mu})\overline{\mu}\right]^{1/2}}{\sqrt{\mathrm{e}}\left[(\overline{W}-\overline{\mu}+1)(\overline{\mu}+1)\right]^{1/2}}\exp\left\{-\frac{Z}{\lambda_\Delta} + \frac{1}{4}\left[\frac{1}{\overline{W}-\overline{\mu}+1} + \frac{1}{\overline{\mu}+1}\right]\right\}. \tag{49}$$

Then we apply the law of conservation of carriers (n is a charge-carrier density) to this expression:

$$n = \frac{2}{\beta V}\sum_{\boldsymbol{k}\Omega} G(\boldsymbol{k}, \Omega)\mathrm{e}^{\mathrm{i}\Omega 0^+}. \tag{50}$$

After carrying out the integration over $\boldsymbol{k}$ and Ω in this equation, with account taken of the electron–hole asymmetry, we derive

$$\overline{\mu} = Z\overline{n} \tag{51}$$

where $\overline{\mu} = \mu/\omega_0$, $\overline{n} = n/(2N_0\omega_0)$.

Self-consistent study of (49) and (51) allows us to obtain the dependence of the quantity T_C on the charge-carrier density $\overline{n}$. Condition (48), which takes account of the imaginary parts of the vertex functions, is an equation for determining the cut-off parameter of the electron–phonon interaction Q_c for a given n.

In the two-dimensional case, inserting (19) in (48) we derive the equation

$$aQ_c^4 - bQ_c^2 - d = 0 \tag{52}$$

where $m = 2\omega_0/E = 1/(2\overline{n})$ and

$$a = \frac{29}{45}\left(\frac{2}{m}\right)^2\left[\frac{A_1(0,\omega_0)}{\omega_0} - \omega_0 B_1(0,\omega_0)\right]$$
$$b = \frac{5}{3m}C_1(0,\omega_0,\mu) \tag{53}$$
$$d = \frac{A_1(0,\omega_0)}{\omega_0}.$$

From definitions (53) the condition $a \ll b$ follows, and as we are looking for the solution with $Q_c^2 \ll 1$ we can present solutions to (52) in the form

$$Q_c^2 \approx -\frac{d}{b}. \tag{54}$$

As $b < 0$, Q_c is a real quantity at $d > 0$. The latter is valid at $\mu < W/2$ (21). So, superconductivity is possible in the two-dimensional system considered, with strong electron correlations, at $\mu < W/2$. At $\mu > W/2$ nonphysical solutions arise (the parameter Q_c becomes an imaginary quantity). This fact allows us to reach the conclusion that for this range of the charge-carrier density, superconductivity is absent ($T_C = 0$).

In the case of a three-dimensional system we insert expression (29) in (48) and as a result obtain

$$a_1 Q_c^4 - b_1 Q_c^2 + d_1 = 0 \tag{55}$$

or

$$Q_c^2 \approx \frac{d_1}{b_1}$$

where

$$a_1 = \frac{5}{6}\left(\frac{2}{m}\right)^2\left[\frac{A_1(0,\omega_0)}{\omega_0} - \omega_0 B_1(0,\omega_0)\right]$$
$$b_1 = \frac{2}{m}C_1(0,\omega_0) \tag{56}$$
$$d_1 = \frac{A_1(0,\omega_0)}{\omega_0} = d.$$

Here the values of Q_c are real quantities if $d < 0$, and as a result superconductivity in the three-dimensional system with strong electron correlations exists over the range $W/2 < \mu < W$. So, we can draw the following general conclusion. There is a value of n_{cr}, at which superconductivity disappears and appears, in a two-dimensional and three-dimensional system, respectively. On the basis of formulae (51) and (35), for $\mu = W/2$ we derive

$$n_{cr} = \frac{\overline{W}/2}{1 + \lambda\left[1 + \lambda P_V^{cr}(Q_c,0,\omega_0)\right]\overline{W}/(\overline{W}+2)} \tag{57}$$

where $P_V^{cr}(Q_c,0,\omega_0) = P_V(Q_c,0,\omega_0)|_{\mu=W/2}$.

Let us consider now the limit case of a symmetrical filling of an energy band in order to compare our results with those in [13]. To do this we have to make the following substitution: $W - \mu \to E/2$, $\mu \to E/2$. As a result we see that there are some coefficients tending to zero, i.e. $C(0,\omega_0) = 0$, $A_1(0,\omega_0)/\omega_0 = 0$, $\omega_0 B_1(0,\omega_0) = 0$. The coefficients $A(0,\omega_0)/\omega_0$ and $\omega_0 B(0,\omega_0)$ transform into the corresponding expressions of [13] at $Q_c^2 \ll 1$, but $b_1 = (2/m)C_1(0,\omega_0) \neq 0$ (21) is absent in [13]. In this case equation (55) has the form $b_1 Q_c^2 = 0$. Because $b_1 \neq 0$ we should put $Q_c = 0$. Neglecting these terms (supposing $b_1 = 0$),

Q_c in [13] seems to be able to take arbitrary values and is considered a theoretical parameter. In fact, strictly speaking, at $Q_c \neq 0$ in a three-dimensional system with a symmetrical filling of an energy band, superconductivity is absent because the quantity ξ_c determining the temperature of the superconducting transition is a complex quantity.

For the coefficient of the isotope effect, on the basis of (49) we have

$$\alpha = -\frac{\mathrm{d}\ln T_C}{\mathrm{d}\ln M} = \frac{1}{2}\left[1 + \frac{\mathrm{d}\ln T_C/\omega_0}{\mathrm{d}\ln\omega_0}\right] \tag{58}$$

where

$$\begin{aligned}
\frac{\mathrm{d}\ln T_C/\omega_0}{\mathrm{d}\ln\omega_0} &= -\frac{1}{4}\left[\frac{\overline{W}-\overline{\mu}+2}{(\overline{W}-\overline{\mu}+1)^2} + \frac{\overline{\mu}+2}{(\overline{\mu}+1)^2}\right] - \frac{1}{\lambda_\Delta}\frac{\mathrm{d}Z}{\mathrm{d}\ln\omega_0} + \frac{Z}{\lambda_\Delta^2}\frac{\mathrm{d}\lambda_\Delta}{\mathrm{d}\ln\omega_0} \\
\frac{\mathrm{d}Z}{\mathrm{d}\ln\omega_0} &= -\frac{1}{2}\left\{\frac{\lambda^2}{2}\left[\frac{\overline{\mu}}{(\overline{\mu}+1)^2+1} + \frac{\overline{W}-\overline{\mu}}{(\overline{W}-\overline{\mu}+1)^2+1}\right]\right. \\
&\qquad \left.\times\left[\frac{\overline{W}-\overline{\mu}}{\overline{W}-\overline{\mu}+1} + \frac{\overline{\mu}}{\overline{\mu}+1}\right] + \lambda_z\left[\frac{\overline{W}-\overline{\mu}}{(\overline{W}-\overline{\mu}+1)^2} + \frac{\overline{\mu}}{(\overline{\mu}+1)^2}\right]\right\} \\
\frac{\mathrm{d}\lambda_\Delta}{\mathrm{d}\ln\omega_0} &= -\frac{3\lambda^2}{2}\left[\frac{\overline{\mu}}{(\overline{\mu}+1)^2+1} + \frac{\overline{W}-\overline{\mu}}{(\overline{W}-\overline{\mu}+1)^2+1}\right].
\end{aligned} \tag{59}$$

In deriving the formulae (58), (59), we have omitted the terms containing Q_c because the contribution of this parameter is negligible at low values.

4. Numerical calculations and discussion of results

We have discussed the influence of a non-adiabaticity effect on the temperature of the superconducting transition T_C and on the coefficient of the isotope effect in quasi-two-dimensional and three-dimensional systems. The quantity T_C is determined by formulae (49) and (51), and the coefficient of the isotope effect α by (58) and (59). We have found a surprising singularity in the behaviour of these quantities: in two-dimensional systems, superconductivity appears at low densities of charge carriers $\overline{n}$ and disappears at n_{cr} ($\mu = W/2$), which corresponds to a symmetrical filling of an energy band. In a three-dimensional system, on the other hand, superconductivity appears at $n = n_{cr}$ ($\mu = W/2$) and disappears at $\mu = W$. Therefore, if we assume that the distribution of carriers is becoming spatially homogeneous in the system with the increase of the charge-carrier density, then the transition from a two-dimensional to a three-dimensional system occurs over the whole field of values $0 < \mu < W$. In this way, all quantities that appear in the definitions of T_C and α ($Z, \lambda_\Delta, \lambda_z$) at $n < n_{cr}$ can be determined from the formulae for a two-dimensional system and those appearing at $n > n_{cr}$ can be determined from the formulae for a three-dimensional one. If the transition does not take place, we should conclude that the feasibility of superconductivity in systems with strong electron correlations ($Q_c \ll 1$) is governed by the dimensionality of the system.

The quantity Q_c is not a constant quantity and is dependent on the charge-carrier density. At the point $n = n_{cr}$ we have $Q_c = 0$. The dependence of Q_c as a function of $\overline{n}$ at various values of λ and W is shown in figure 1. Curves labelled by numbers without primes denote the results for a two-dimensional system, and those labelled by numbers with primes denote the results for a three-dimensional system. The vertical dashed lines in this and the other figures correspond to the boundaries between the fields of superconductivity in two-dimensional and three-dimensional systems. The intersection of such a line with the abscissa gives the critical value of the parameter $\overline{n}$ (n_{cr}) at which superconductivity disappears in a two-dimensional system and is induced in a three-dimensional system. The quantity Q_c decreases with the

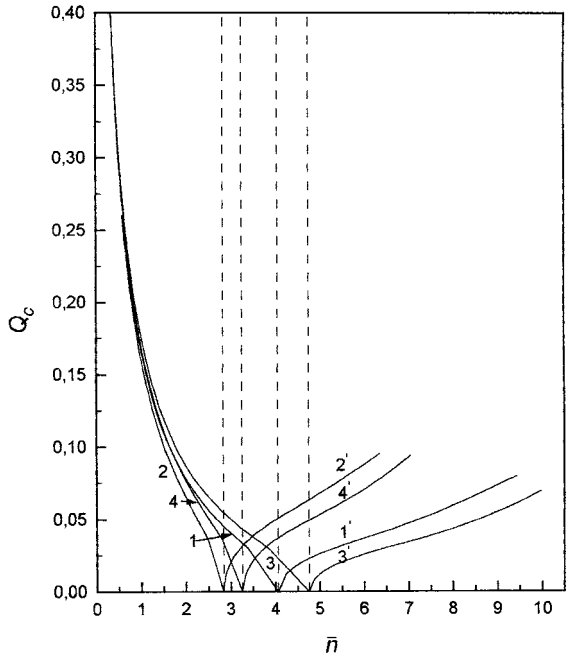


Figure 1. The dependence of the cut-off momentum Q_c on the charge-carrier density $\overline{n}$. Curves $1, 1'$ correspond to $\lambda = 0.5$, $\overline{W} = 6$; curves $2, 2'$ correspond to $\lambda = 0.5$, $\overline{W} = 4$; curves $3, 3'$ correspond to $\lambda = 0.3$, $\overline{W} = 6$; curves $4, 4'$ correspond to $\lambda = 0.3$, $\overline{W} = 4$.

increase of $\overline{n}$ and approaches zero at $n = \overline{n}_{cr}$ in a two-dimensional system and increases slightly (from the value $Q_c = 0$) with increase of $\overline{n}$ in a three-dimensional system. These values of Q_c, obtained as a result of the solutions for (52), (55) and (51), are used further to calculate the dependence of the temperature of the superconducting transition on $\overline{n}$ on the basis of formulae (49), (51) and others which determine the quantities appearing in (49) and (51).

Figure 2 shows this dependence: the ratio T_C/ω_0 increases with the increase of $\overline{n}$ for a two-dimensional system and decreases for a three-dimensional one. There we observe the essential dependence of T_C/ω_0 on the parameters λ and W. It follows from this figure that the effects of non-adiabaticity increase T_C/ω_0 (at the values of $\lambda = 0.5$ and 0.3 considered) approximatively three times as much as that for ordinary superconductors (compare $1, 1'$ and $2, 2'$ with curve 5, and also $3, 3'$ and $4, 4'$ with curve 6). So, if we apply a theory for describing superconductivity in oxide ceramics, we can come to the following conclusion. Upon doping a system with an oxygen or non-isovalent impurity, which favours increase of the charge-carrier density, it is possible to obtain the values of T_C characteristic of these materials even at intermediate values of the interaction coupling λ, due to the effect of non-adiabaticity. Taking account of other singularities of high-T_C materials—for example, overlapping of energy bands [4–6] as well as the effect of non-adiabaticity—can create more favourable conditions for a higher T_C.

Figure 3 shows the dependence of the coefficient of the isotope effect α on the charge-carrier density: with the increase of $\overline{n}$ in a two-dimensional system, α increases, and in a three-dimensional one, it decreases. The coefficient α depends on λ and W in the same way as T_C. Therefore, low values of α are possible at low $\overline{n}$ in a two-dimensional system and at high $\overline{n}$ in a three-dimensional one. For the range of the maximum values of T_C for the theoretical

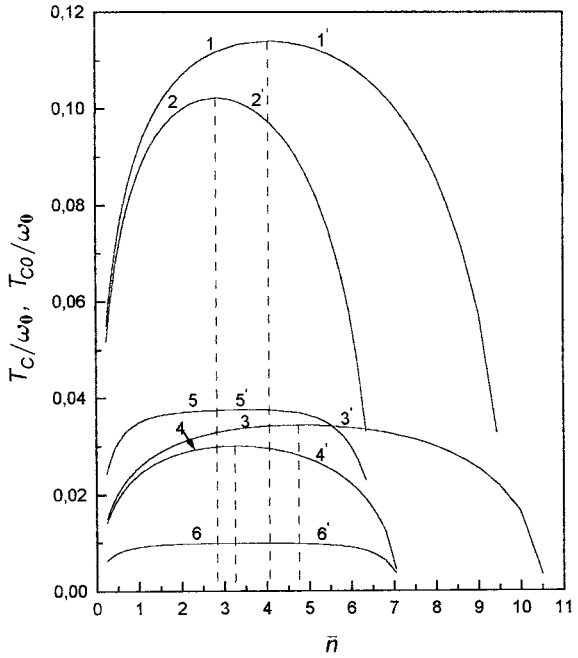


Figure 2. The dependence of the temperature of the superconducting transition T_C on the charge-carrier density $\overline{n}$. Curves 1–4, 1′–4′ correspond to T_C and the values of λ and $\overline{W}$ for these curves coincide with those for figure 1. Curves 5, 5′ and 6, 6′ correspond to T_{C0} with $\lambda = 0.5$, $\overline{W} = 4$ and $\lambda = 0.3$, $\overline{W} = 4$, respectively.

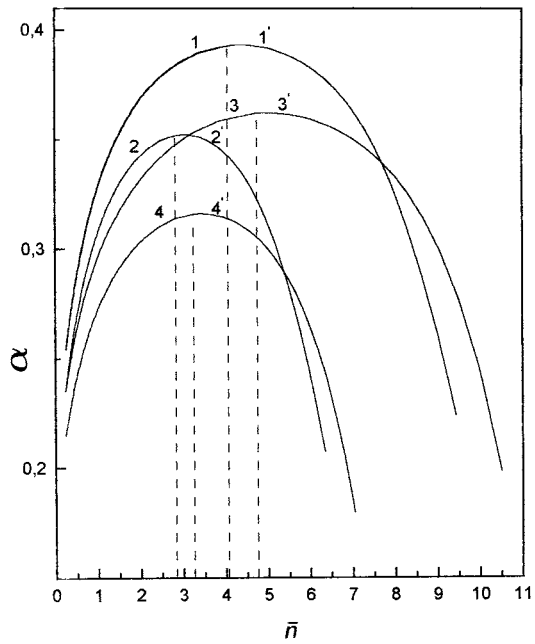


Figure 3. The dependence of the coefficient of the isotope effect α on the charge-carrier density $\overline{n}$. The labelling of the curves corresponds to that for figure 1.

parameters given above, we obtain $\alpha \sim 0.3$–0.4. These values are higher than those observed experimentally for some oxide ceramics. In the case of a narrow energy band ($W \sim \omega$), the coefficient α can achieve values ~ 0.2, which is in accordance with experimental data. These results confirm the non-phonon essence of a superconductivity pairing mechanism in high-T_C materials ($W \gg \omega_0$). The above-suggested non-adiabatic theory for T_C-determination is stated above to be valid for both electron–phonon and non-phonon mechanisms of superconductivity at low values of the transferred momentum $q \ll 2k_F$.

Figure 4 shows the dependences of T_C/ω_0 and α on the parameter $1/(2n_{cr})$ for a three-dimensional case at $Q_c = 0$. It corresponds totally to the results in [13] for low values of Q_c ($Q_c \ll 1$) taking into consideration the dependence of these quantities on the Migdal parameter ($=m|_{\mu=W/2} = 1/(2n_{cr})$) in systems with half-filled energy bands. The analysis of this result shows that low values of α, near half-filling of the energy band with the maximum T_C, can be obtained for a system with narrow energy bands ($\overline{W} \sim 1$; bandwidth W is of the same order as the Debye frequency ω_0).

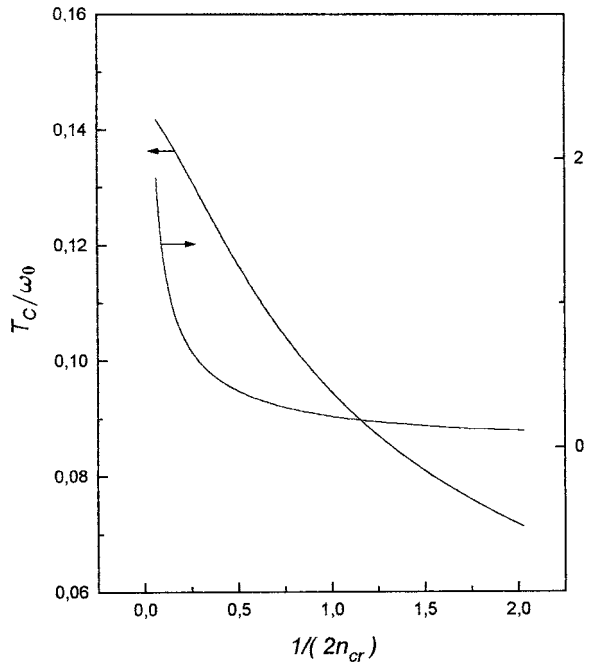


Figure 4. The dependence of the temperature of the superconducting transition T_C and of the coefficient of the isotope effect α on the parameter $1/(2n_{cr})$ which corresponds to the Migdal parameter $m = (\omega_0/E_F)|_{\mu=W/2}$ for systems with half-filled energy bands.

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